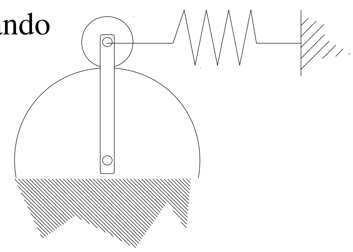
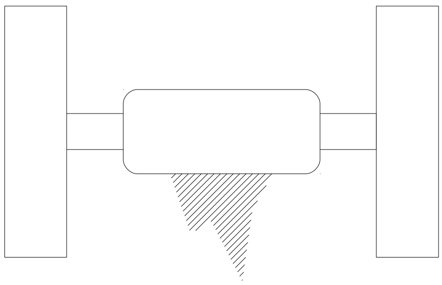
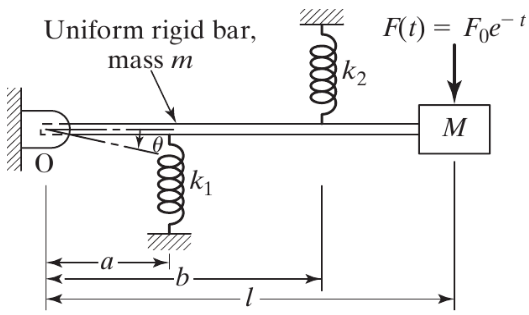
1) Calcule a frequência natural do sistema mostrado ao lado, considerando que que o disco maior é fixo e tem raio *R*, o disco menor tem raio *r* e massa *m*, e a barra tem massa m_b, e a mola tem rigidez *k*. *(Valor 3,0 pontos)*



2) Na figura, os discos representam engrenagens rígidas que tem 100mm de diâmetro e 15mm de largura de face. Elas estão conectadas entre si por árvores com diâmetro igual 15mm e comprimento igual a 350mm. As árvores giram livremente sobre os mancais mostrados, que impedem qualquer movimento axial. Todos os componentes são feitos de aço, com módulo de elasticidade igual a 210 GPa e módulo de cisalhamento igual a 80GPa. Diga quantos graus de liberdade tem o sistema e quais as suas frequências naturais. A relação entre o ângulo de torção e o momento aplicado em uma barra de seção circular constante é dado por $M = GJ\,\theta / l$, e o momento de inércia da seção é $\pi d^4/32$ e o momento de inércia de massa de um cilindro é $mD^2/8$. *(Valor 4.0 pontos)*



3) Calcule a resposta da barra mostrada na figura ao lado, sabendo que $k_1 = k_2 = 5\,kN/m$, $k_1 = k_2 = 5\,kN/m$, $k_1 = k_2 = 5\,kN/m$, $a = 0.25\,m$, $b = 0.45\,m$, $l = 1\,m$, $b = 0.45\,m$, $M = 50\,kg$, $m = 12\,kg$ e $F_0 = 500\,N$. *(Valor 3.0 pontos)*



$\omega = 2\pi f$ $\quad f = \frac{1}{\tau}$ $\quad T = \frac{1}{2} m \dot{x}^2, \quad T = \frac{1}{2} J_0 \dot{\theta}^2, \quad U = \frac{1}{2} \kappa x^2, \quad U = \frac{1}{2} F x$ $\quad k_t = \frac{GJ}{L}$

$\omega_n = \sqrt{\frac{k}{m}}, \quad \omega_n = \sqrt{\frac{k_t}{J_0}}, \quad \omega_d = \sqrt{1-\zeta^2}\,\omega_n, \quad \zeta = \frac{c}{c_c}, \quad c_c = 2 m \omega_n$ $\quad \delta_{st} = \frac{F_0}{k}$

$x(t) = X e^{-\zeta \omega_n t} \cos(\omega_d t - \varphi), X = \frac{\sqrt{x_0^2 \omega_n^2 + \dot{x}_0{}^2 + 2 x_0 \dot{x}_0{}^2 \zeta \omega_n}}{\omega_d}, \omega_d = \sqrt{1-\zeta^2}\,\omega_n, \varphi = \arctan\left(\frac{\dot{x}_0 + \zeta \omega_n x_0}{x_0 \omega_d}\right)$

$x(t) = (C_1 + C_2 t) e^{-\omega_n t}, \quad C_1 = x_0, \quad C_2 = \dot{x}_0 + \omega_n x_0$ $\quad \beta = \frac{h}{k}$ $\quad \delta = \pi \beta$ $\quad x(t) = \frac{1}{m \omega_d} \int_0^t F(\tau) e^{-\zeta \omega_n (t-\tau)} \sin \omega_d (t-\tau) \; d\tau$

$x(t) = C_1 e^{(-\zeta + \sqrt{\zeta^2 - 1})} \omega_n t + C_2 e^{(-\zeta - \sqrt{\zeta^2-1})} \omega_n t, \quad C_1 = \frac{x_0 \omega_n (\zeta + \sqrt{\zeta^2 - 1}) + \dot{x}_0}{2 \omega_n \sqrt{\zeta^2 - 1}}, \quad C_2 = \frac{-x_0 \omega_n (\zeta - \sqrt{\zeta^2 - 1}) - \dot{x}_0}{2 \omega_n \sqrt{\zeta^2 - 1}}$

$\delta = \frac{1}{n} \ln\left(\frac{x_1}{x_{n+1}}\right), \quad \delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}, \quad \delta = 2\pi\zeta \text{ para } \zeta \ll 1$ $\quad \Delta W = \pi \omega c X^2$ $\quad \Delta W = \pi h X^2$

$T_d = \frac{X}{Y} = \left(\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right)^{\frac{1}{2}}$ $\quad \frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$ $\quad \frac{M x}{m e} = r^2 |H(i\omega)|, \varphi = \arctan\left(\frac{2\zeta r}{1-r^2}\right)$

$c = \sqrt{\frac{P}{\rho}} \quad c = \sqrt{\frac{E}{\rho}} \quad c = \sqrt{\frac{G}{\rho}} \quad c = \sqrt{\frac{EI}{\rho A}} \quad \alpha \tan \alpha = \beta \quad \alpha = \frac{\omega l}{c} \quad \beta = \frac{m}{M}$ $\quad \omega = 2\pi f$ $\quad f = \frac{1}{\tau}$

$L[\ddot{x}(t)] = s^2 X(s) - s x(0) - \dot{x}(0)$ $\quad L[\dot{x}(t)] = s X(s) - x(0)$ $\quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = Q_j^n, \quad j = 1,2,\ldots,n$

$T = \frac{1}{2} m \dot{x}^2, \quad T = \frac{1}{2} J_0 \dot{\theta}^2, \quad U = \frac{1}{2} \kappa x^2, \quad U = \frac{1}{2} F x$ $\quad Z_{rs}(i\omega) = -\omega^2 m_{rs} + i \omega c_{rs} + k_{rs}$ $\quad \boldsymbol{Z}(i\omega) \boldsymbol{X} = \boldsymbol{F_0}$

$\omega_n = \sqrt{\frac{k}{m}}, \quad \omega_n = \sqrt{\frac{k_t}{J_0}}, \quad \omega_d = \sqrt{1-\zeta^2}\,\omega_n, \quad \zeta = \frac{c}{c_c}, \quad c_c = 2 m \omega_n$ $\quad \delta_{st} = \frac{F_0}{k}$ $\quad \boldsymbol{X} = \boldsymbol{Z}(i\omega)^{-1} \boldsymbol{F_0}$

$\boldsymbol{Z}(i\omega) = \begin{bmatrix} Z_{11}(i\omega) & Z_{12}(i\omega) \\ Z_{12}(i\omega) & Z_{22}(i\omega) \end{bmatrix}$ $\quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $\quad \delta = \frac{1}{n} \ln\left(\frac{x_1}{x_{n+1}}\right), \quad \delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}, \quad \delta = 2\pi\zeta \text{ para } \zeta \ll 1$

$\frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$ $\quad H(i\omega) = \frac{1}{(1-r^2) + i 2 \zeta r}, |H(i\omega)| = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$